

Self-Similarity and Power Laws

Consider function $G(r)$ that is *lacking* any special length scale.

$$\begin{aligned}r &\rightarrow \lambda r \\ G(\lambda r) &= \phi(\lambda, r)G(r)\end{aligned}$$

- λ dimensionless, ϕ dimensionless
- *no* parameter available that could make r dimensionless
- $\Rightarrow \phi$ depends only on λ

$$\begin{aligned}\phi(\lambda\mu) &= \phi(\lambda)\phi(\mu) && (d/d\mu) \\ \lambda\phi'(\lambda\mu) &= \phi(\lambda)\phi'(\mu) && (\mu = 1, \phi'(1) =: q) \\ \lambda\phi'(\lambda) &= \phi(\lambda)q \\ \frac{\phi'(\lambda)}{\phi(\lambda)} &= \frac{q}{\lambda} && \left(\int d\lambda, \phi(1) = 1\right) \\ \ln \phi(\lambda) &= q \ln \lambda \\ \phi(\lambda) &= \lambda^q\end{aligned}$$

$$\begin{aligned}G(\lambda r) &= \lambda^q G(r) && (\lambda = r_0/r) \\ G(r_0) &= \left(\frac{r_0}{r}\right)^q G(r) \\ G(r) &= \left(\frac{r}{r_0}\right)^q G(r_0)\end{aligned}$$