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Bénard-Marangoni instability in a viscoelastic ferrofluid

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Abstract. In this manuscript we report theoretical and numerical results on convection for a magnetic fluid in a viscoelastic carrier liquid. The viscoelastic properties is given by the Oldroyd model. We impose the lower interface to be rigid, whereas the upper one free which is assumed to be non-deformable and flat. Also, at the upper interface the surface tension is taken to vary linearly with the temperature. Using a spectral method we calculate numerically the convective thresholds for both stationary and oscillatory bifurcation. The effect of the viscoelasticity and the Kelvin force on instability thresholds are emphasized.

1 Introduction

Ferrofluids are magnetic stable colloidal suspensions of magnetic nanoparticles dispersed in a carrier liquid. In the absence of an external magnetic field the magnetic moments of the particles are randomly orientated and there is no net macroscopic magnetization. In an external magnetic field, however, the particles' magnetic moments easily orient and a large (induced) magnetization is present. There are two additional features in ferrofluids not found in ordinary fluids, the Kelvin force and the body couple [1]. In addition, in an external magnetic field, a ferrofluid exhibits additional rheological properties such as a field-dependent viscosity, special adhesion properties, and a non-Newtonian behavior, among others [2]. Convection in ferrofluids has been a topic of great interest In the last decades. In addition, heat transfer through magnetic fluids has been of outstanding technological importance and was therefore also a leading area of scientific studies [3].

The first macroscopic description of magnetic fluids was given by Neuringer and Rosensweig [4]. The convective instability of a magnetic fluid layer heated from below in the presence of a uniform vertical magnetic field was discussed later by Finlayson [5]. Both cases, shear free and rigid horizontal boundaries, were investigated within the linear stability method. Ryskin and Pleiner [6], using a generalized hydrodynamic description based on non-equilibrium thermodynamics, derived a complete set of partial differential equations to describe ferrofluids in an external magnetic field. Other works in ferrofluid convection can be found in Refs. [8–12]

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A popular way to describe the viscoelastic properties of fluids is the use of a constitutive equation, which relates the stress and strain rate tensors. Rheology is the science devoted to generalize the linear, static Newtonian relation to the various, more complicated cases of non-Newtonian behavior. Very often an Oldroyd constitutive equation [13] is employed to realistically describe viscoelastic properties. In this model, the stress tensor is basically decomposed into both a polymeric-like elastic contribution and a Newtonian-like solvent contribution. Convection in such "Oldroyd fluids" has been studied by various authors for different physical and geometrical cases, e.g. both for free-free or rigid-rigid boundary conditions [14, 15]. By heating a fluid layer from below, as a general result, oscillatory modes can be obtained at onset in competition to the usual stationary convection states. Which type of instability, stationary or oscillatory, appears first, depends on the values of the rheological parameters. Recently, thermal convection in viscoelastic magnetic fluids was studied for idealized and rigid boundary conditions [16–19].

The Marangoni instability considered in this manuscript is the standard example for a surface tension driven instability. If a temperature gradient is applied from below to a fluid layer with an upper free surface, temperature fluctuations result in surface stress fluctuations due to the temperature dependence of the surface tension. The latter either decay or have to be compensated by flow, thus rendering the quiescent heat conducting state unstable to convection above a critical temperature gradient. The linear analysis of the convection in a magnetic fluid with a deformable free surface was studied by Weilepp and Brand [20] and by Hennenberg *et al.* [21]. The linear and weakly nonlinear analysis in the case of viscoelastic non-magnetic fluids was performed for a non-deformable free surface by Lebon *et al.* [22, 23]. The Marangoni problem of Newtonian ferrofluids has been studied for different situations in Refs. [24–28]. The eigenvalues and eigenfunctions of the adjoint problem and adjoint boundary conditions for the case of a deformable free surface for the Marangoni problem have been derived only recently [29].

The purpose of the present work is to analyze the influence of the viscoelasticity in Bénard-Marangoni convective thresholds in magnetic fluid, in particular, where the separation ratio and magnetic separation ratio are not too large the simple fluid approximation can be used [6]. To this aim an Oldroyd viscoelastic magnetic fluids heated from below is considered. The description of the system involves many parameters whose values have not yet been determined accurately. Therefore, we are left with some freedom in fixing the parameter values. Since the boundary condition are complicated, we numerically solve the linearized system using a collocation spectral method in order to determine the eigenfunctions and eigenvalues and consequently the convective thresholds. The paper is organized as follows: In Section 2, the basic hydrodynamic equations for viscoelastic magnetic fluid convection are presented. In Section 3 the linear stability analysis is performed. Finally, conclusions are presented in Section 4.

2 Basic equations

We consider a layer of an incompressible, viscoelastic, and magnetic fluid of thickness d, with very large horizontal (xy-plane) extension subject to a vertical gravitational field $\mathbf{g}$ and temperature gradient. The magnetic fluid properties can be modeled as electrically nonconducting superparamagnets. The external magnetic field $\mathbf{H}$ is assumed also to be vertical (parallel to the $\hat{\mathbf{z}}$ axis). It would be homogeneous, if the magnetic fluid were absent. Let us choose the z-axis such that $\mathbf{g} = -g\hat{\mathbf{z}}$ and that the layer has its interfaces at coordinates $z = -d/2$ and $z = d/2$. We impose the lower interface to be rigid and the upper one free. The latter one is commonly assumed to be

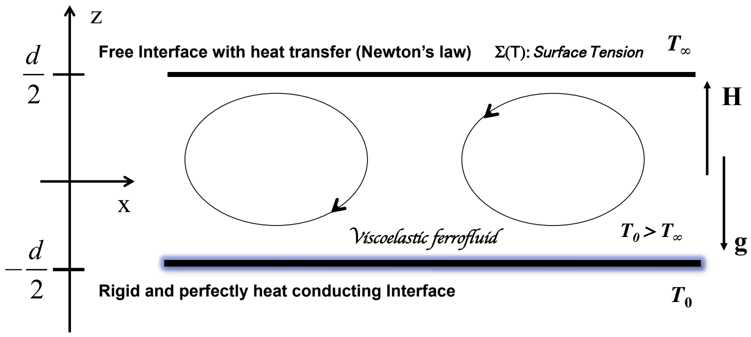

Fig. 1. A vertical cut through the fluid layer. Note the y-axis point into the xz-plane.

non-deformable and flat [22,23,27]. This is a reasonable approximation if the thickness d is not too small, for example in the case of aqueous suspensions $d > 10^{-3}m$ [30]. At the upper, free interface, the surface tension Σ is taken to vary linearly with the temperature, T, that means $\Sigma = \Sigma_0 - \gamma_T \triangle T$, such that γ_T is commonly a positive constant and where $\triangle$ denotes deviations from the ground state. The set-up of the problem is drawn in Fig. (1). Under the Boussinesq approximation, the dimensionless balance equations of the perturbation of the conductive state read as [18,19,29]

$$\nabla \cdot \mathbf{v} = \mathbf{0} \tag{1}$$

$$P^{-1} d_t \mathbf{v} = -\nabla p_{eff} + \nabla \cdot \overline{\boldsymbol{\tau}} + Ra\boldsymbol{\Xi} \tag{2}$$

$$(1 + \Gamma D_t)\overline{\boldsymbol{\tau}} = (1 + \Gamma \Lambda D_t)\overline{\boldsymbol{D}} \tag{3}$$

$$d_t(\theta - M_4 \partial_z \phi) = (1 - M_4)w + \nabla^2 \theta \tag{4}$$

$$(\partial_{zz} + M_3 \nabla_\perp^2)\phi - \partial_z \theta = 0 \tag{5}$$

$$\nabla^2 \phi_{ext} = 0 \tag{6}$$

where $\{\mathbf{v}, \overline{\boldsymbol{\tau}}, \theta, \phi\}$ are the dimensionless velocity perturbation, the dimensionless extra stress tensor, the temperature perturbation and the dimensionless magnetic potential perturbation, respectively. Here $d_t f = \partial_t f + \mathbf{v} \cdot \nabla f$ is the total derivative, p_{eff} is the dimensionless effective pressure which contains the static hydrodynamic pressure, and $\boldsymbol{\Xi} = \Pi_1(\theta, \phi)\hat{\mathbf{z}} + M_1 \theta \nabla(\partial_z \phi)$ with $\Pi_1 = (1 + M_1)\theta - (M_1 - M_5)\partial_z \phi$ and $\nabla_\perp^2 = \partial_{xx} + \partial_{yy}$. In Eqs. (1)-(6), the following groups of dimensionless numbers have also been introduced: ***(a)*** (pure fluids) The Rayleigh number, $Ra = \alpha_T g \beta d^4/\kappa\nu$, accounting for buoyancy effects; and the Prandtl number, $P = \nu/\kappa$, relating viscous and thermal diffusion time scales. ***(b)*** (magnetic fluid) The strength of the magnetic force relative to buoyancy is measured by the parameter $M_1 = \beta \chi_T^2 H_0^2/(\rho_0 g \alpha_T (1 + \chi))$; the nonlinearity of the magnetization, $M_3 = (1 + \chi)/(1 + \chi + \chi_H H_0^2)$, a measure of the deviation of the magnetization curve from the linear behavior $M_0 = \chi H_0$ with $M_3 \gtrsim 1$ [5]; the relative strength of the temperature dependence of the magnetic susceptibility $M_4 = \chi_T^2 H_0^2 T_0/c_H(1 + \chi)$; and the ratio of magnetic variation of density with respect to thermal buoyancy $M_5 = \alpha_H \chi_T H_0^2/(\alpha_T(1 + \chi))$. ***(c)***(viscoelastic fluid) The Deborah number, $\Gamma = \lambda_1 \bar{\kappa}/d^2$, and the ratio between retardation and stress relaxation times, $\Lambda = \lambda_2/\lambda_1$.

In these dimensionless numbers different physical quantities appear such as ρ_0 the reference mass density, c_H the specific heat capacity at constant volume and magnetic field, T_0 the reference temperature, H_0 the reference magnetic field, χ_T the pyromagnetic coefficient, κ the thermal diffusivity, χ_H the longitudinal magnetic susceptibility, α_T the thermal expansion coefficients and α_H the magnetic expansion coefficients, ν the static viscosity, λ_1 is the relaxation time, and λ_2 is the retardation time. The last two parameters characterize the viscoelastic time scales which for thermodynamic stability reasons both, λ_1 and λ_2, are taken to be positive. Also, $\beta = (T_0 - T_\infty)/(d + (\epsilon/h))$ where T_∞ is the uniform temperature in the air layer above the liquid, $\epsilon = \kappa c_H$ is the heat conductivity of the liquid, and h is the heat transfer coefficient. The symbol D_t in Eq. (3) denotes an invariant ("frame-indifferent") time derivative, defined as

$$D_t \overline{\boldsymbol{\tau}} = d_t \overline{\boldsymbol{\tau}} + \overline{\boldsymbol{\tau}} \cdot \overline{\mathbf{W}} - \overline{\mathbf{W}} \cdot \overline{\boldsymbol{\tau}} + a(\overline{\boldsymbol{\tau}} \cdot \overline{\mathbf{D}} + \overline{\mathbf{D}} \cdot \overline{\boldsymbol{\tau}}), \tag{7}$$

where $\overline{\mathbf{D}}$ and $\overline{\mathbf{W}}$ are the symetric and skew-symmetric part of the velocity field gradient, respectively. Also, a is a phenomenological parameter that lies in the range -1 to $+1$. For $a = -1$, one gets the lower convected Jeffrey's model (Oldroyd B), for $a = 0$ one gets the so-called corotational Jeffrey's model, and $a = 1$ describes the upper convected Jeffrey's model (Oldroyd A). Let us comment that the coefficient a is not completely independent of the other rheological parameters [31]. Some limiting cases are $\lambda_2 = 0$ that leads to a Maxwellian fluid, while a Newtonian fluid requires both $\lambda_1 = 0$ and $\lambda_2 = 0$.

Let us comment on the numerical values of the parameters; the Rayleigh number Ra can be changed by several orders of magnitude by varying the applied temperature gradient, with $Ra \sim 10^2 - 10^3$ relevant in the present case. A typical value for P in viscoelastic fluids is $P \sim 10^0 - 10^3$ with $P \sim 10$ for aqueous systems. The magnetic numbers are field dependent with $M_1 \sim 10^{-4} - 10$, $M_3 \gtrsim 1$, $M_4 \sim M_5 \sim 10^{-6}$ for typical magnetic field strengths [6] M_1 is directly proportional to H_0^2, while M_3 is only a weak function of the external magnetic field. Since M_4 and M_5 are very small and not related to viscoelastic effects, which we are interested in here, we expect not to loose any reasonable aspect of the problem under consideration by putting them to zero. Kolodner [32] and the group of Chu [33–35] have suggested that for aqueous suspensions the Deborah number is about $\Gamma \sim 10^{-3} - 10^{-1}$, but for other kinds of viscoelastic fluids the Deborah number can be of the order of $\Gamma \sim 10^3$. Unfortunately, no experimental data are available for either the retardation or the stress relaxation times, so we treat Λ as arbitrary in the range $[0, 1]$. In the next section, we study the stability of the conduction state.

3 Linear stability analysis

In order to calculate the linear stability, we only need the linear parts of Eqs. (1)-(5). This is readily done by neglecting the advective terms $\mathbf{v} \cdot \nabla$ and replacing D_t by ∂_t. Moreover, the effective pressure and two components of the velocity field could also easily be eliminated by applying the curl ($\nabla \times ...$) and double curl ($\nabla \times \nabla \times ...$) of the Navier-Stokes equation and then considering only the z-component of the resulting equations, w (i.e. the vertical velocity component). After some algebra, the linear equations read

$$P^{-1} \partial_t \nabla^2 w = \nabla^2 (\nabla \cdot \overline{\boldsymbol{\tau}})_z + Ra \nabla_\perp^2 \mathcal{L}_\Xi \tag{8}$$

$$(1 + \Gamma \partial_t)(\nabla \cdot \overline{\boldsymbol{\tau}})_z = (1 + \Gamma \Lambda \partial_t) \nabla^2 w \tag{9}$$

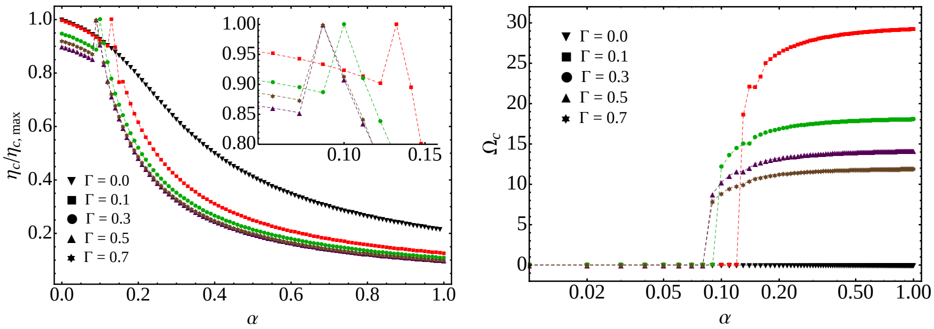

Fig. 2. η_c (left) and Ω_c (right) as a function of α for different values of Γ (black down-triangles $\Gamma = 0.0$, red squares $\Gamma = 0.1$, green dots $\Gamma = 0.3$, purple up-triangles $\Gamma = 0.5$, brown stars $\Gamma = 0.7$). The fixed parameters are $P = 10$, $\Lambda = 0.5$, $M_1 = 10$, $M_3 = 1.1$, $Bi = 10^{-6}$, and $\chi_b = 1$. The inset on η_c shows a magnification in the stationary−oscillatory transition.

$$\partial_t \theta = w + \nabla^2 \theta \tag{10}$$

$$(\partial_{zz} + M_3 \nabla_\perp^2)\phi - \partial_z \theta = 0 \tag{11}$$

where $\mathcal{L}_\Xi = (1+M_1)\theta - M_1 \partial_z \phi$. We remark that Eqs. (8) and (9) can be combined in order to get a single equation for w. One can define the vector field $\mathbf{u} = (\theta, \phi, w)^T$ that contains the important variables for the linear analysis. Using standard techniques, the spatial and temporal dependencies of $\mathbf{u}$ are separated using normal mode expansion

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{U}(z) \exp[i\mathbf{k} \cdot \mathbf{r}_\perp + st], \tag{12}$$

being $\mathbf{U} = (\Theta, \Phi, W)^T$, where $\mathbf{k}$ is the horizontal wavenumber vector of the perturbations, $\mathbf{r}_\perp$ is the horizontal vector position and where $s = \sigma + i\Omega$ denotes the complex eigenvalues; σ is the growth factor of the perturbation, and Ω its frequency. Using the ansatz (12), Eqs. (8)-(11) are reduced to the following coupled ordinary differential equations

$$D^2 \Theta = \xi_1 \Theta - W \tag{13}$$

$$D^2 \Phi = \xi_2 \Phi + D\Theta \tag{14}$$

$$D^4 W = \xi_3 D^2 W - \xi_4 W + \xi_5 \Theta - \xi_6 D\Phi \tag{15}$$

where $D^n f = \partial_z^n f$, $\xi_1 = k^2 + \sigma$, $\xi_2 = M_3 k^2$, $\xi_3 = 2k^2 + s\mathcal{Q}/P$, $\xi_4 = k^2 \left(k^2 + s\mathcal{Q}/P\right)$, $\xi_5 = Rak^2 \left(1 + M_1\right) \mathcal{Q}$ and $\xi_6 = Rak^2 M_1 \mathcal{Q}$ such that $\mathcal{Q} = (1 + s\Gamma)/(1 + s\Lambda\Gamma)$.

The boundary conditions for a viscous or viscoelastic fluid are at the lower rigid interface ($z = -1/2$)

$$W = DW = \Theta = 0, \tag{16}$$

and at the upper free interface ($z = +1/2$)

$$W = D^2 W + k^2 Ma \mathcal{Q} \Theta = D\Theta + Bi\Theta = 0, \tag{17}$$

where $Ma = \gamma_T \beta d^2/\kappa\nu$ is the Marangoni number. We remark that the Marangoni instability is a capillarity effect and it arises from the variation of the surface tension Σ at the upper surface with temperature. In addition, the Biot number, $Bi = dh/\epsilon$ arises from the Newton's heat transfer law due to cooling at the upper boundary. Note that Bi for a perfectly heat conducting surface tends to infinity, and for an adiabatically

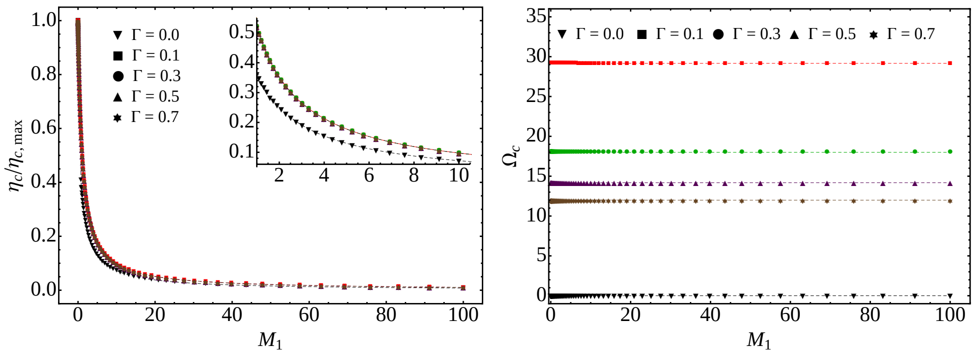

Fig. 3. η_c (left) and Ω_c (right) as a function of M_1 at $\alpha = 1$ for different values of Γ (black down-triangles $\Gamma = 0.0$, red squares $\Gamma = 0.1$, green dots $\Gamma = 0.3$, purple up-triangles $\Gamma = 0.5$, brown stars $\Gamma = 0.7$). The other fixed parameters are $P = 10$, $\Lambda = 0.5$, $M_3 = 1.1$, $Bi = 10^{-6}$, and $\chi_b = 1$. The inset on η_c shows a magnification in a specific range of M_1.

insulated boundary tends to zero. The assumption of a flat upper surface is reasonable for small crispation number $Cr = \nu\kappa/(\Sigma_0 d) < 10^{-3}$ [30].

On the other hand, in the case of a finite magnetic permeability χ_b of the boundaries, the scalar magnetic potential must satisfy the following dimensionless boundary conditions [6, 7]

$$(1 + \chi_b)(D\Phi - \Theta) \pm k\Phi = 0, \tag{18}$$

at $z = \pm 1/2$. The occurrence of temperature variations in the magnetic boundary condition at the free boundary (at $z = -1/2$ there is $\Theta = 0$) is specific for the combination of surface and magnetic effects. The effective surface susceptibility $\chi_b = \chi-(1+\chi)(M_3-1)$ is slightly smaller than the linear one. Note that in the limit when χ_b tends to infinity, Eqs. (18) gives $D\Phi = \Theta$, which is often used as a simplified boundary condition. Finally, let us remark that if we take into account a deformable free surface additional important cross effects, such as the Cowley-Rosensweig instability [21, 25, 26], may be appear. This issue will be included in future works.

Since Ra and Ma are not completely unrelated, one can define two other parameters, η and α which measure better the ratio between the gravity and the capillarity effects [23, 36]. These parameters are called *rate of heating* and *gravity parameter*, respectively. The relationships with the dimensionless numbers are $Ra = \eta\alpha Ra_0$ and $Ma = (1 - \alpha)\eta Ma_0$, such that $Ra_0 = 669$ and $Ma_0 = 79.607$. The inverse relationships are

$$\eta = \frac{Ra}{Ra_0} + \frac{Ma}{Ma_0} \tag{19}$$

and

$$\alpha = \frac{RaMa_0}{MaRa_0 + RaMa_0}. \tag{20}$$

The values Ra_0 and Ma_0 correspond to the critical Rayleigh number obtained in the absence of capillarity for Newtonian fluids and to the critical Marangoni number obtained in the absence of gravity for Newtonian fluids with an adiabatically isolated upper surface, respectively. Note that the case of $\alpha = 0$ corresponds to pure Marangoni instability ($Ra = 0$), while the case $\alpha = 1$ correspond to pure Rayleigh-Bénard instability ($Ma = 0$). In the following, we will calculate, for some fixed values of α, the critical value η_c of η, where the instability sets in.

In order to solve Eqs. (13)-(15) with these realistic boundary conditions, we use a spectral collocation method. Spectral methods ensure an exponential convergence to the solution and are the best available numerical techniques for solving simple eigenvalue – eigenfunction problems. Here, we follow the technique of collocation points on a Chebyschev grid as described by Threfethen [37]. The collocation points (Gauss−Lobatto) are located at height $z_j = \cos(j\pi/N)$ where the index j runs from $j = 0$ to $j = N$. Note that here the z-variable ranges from -1 to $+1$ and one has therefore to rescale Eqs. (13)-(15) accordingly, because the physical domain is defined in the range $(-1/2, +1/2)$. We use $N = 120$ collocation points in the vertical direction, for which the equations and the boundary conditions are expressed. By using the collocation method, the set of differential equations (13)-(15) is transformed into a set of linear algebraic equations. The eigenfunctions $(\Theta(z), \Phi(z), W(z))$ are transformed into eigenvectors defined at the collocation points, $\mathbf{X} = (\Theta_0, ..., W_N)$, such that $\Psi_j = \Psi(z_j)$. After this stage of discretization, one is left with a classical generalized eigenvalue problem, $\bar{\mathbf{A}}\mathbf{X} = \eta\bar{\mathbf{B}}\mathbf{X}$, where η and $\mathbf{X}$ are the eigenvalue and eigenvector, respectively.

In the case of the oscillatory instability considered here, one has to make sure that η (as being a physical quantity) is a real number by choosing a correct value for Ω. Therefore, one is left with a triplet $\{\eta, k, \Omega\}$ that defines a marginal stability condition (for a fixed value of the horizontal wavenumber k). This procedure is repeated for several values of k leading to the marginal stability curve η versus k. The minimum of this curve gives η_c and k_c, and the corresponding value for the critical frequency Ω_c.

Instead of using the linearized Oldroyd model, Eq. (9), one could have used a linear viscoelastic description in terms of a relaxing strain tensor, U_{ij},

$$\partial_t U_{ij} = D_{ij} - \Gamma_1^{-1} U_{ij} \tag{21}$$

where the extra stress tensor $\overline{\tau}_{ij} = p(E_1 U_{ij} + D_{ij})$ contains an elastic part (with the dimensionless elastic modulus E_1) and the usual Newtonian viscosity (which is the p number in dimensionless units). Within the linear domain both descriptions are equivalent with $\Gamma = \Gamma_1$ and $\Lambda = (1 + E_1\Gamma_1)^{-1}$, revealing however that Λ is restricted by $0 < \Lambda < 1$. The static Newtonian viscosity ν_N, used in the viscoelastic description, is related to the asymptotic viscosity ν (used in the Oldroyd case) by $\nu = \nu_N/\Lambda$.

The main advantage of the use of the explicit viscoelasticity, Eq.(21), is that it can easily be generalized in a straightforward physical manner into the nonlinear domain. In addition, for complex liquids that need the introduction of additional degrees of freedom the combination of the strain tensor dynamic equation with those additional degrees of freedom follows standard thermodynamic and hydrodynamic procedures, while the heuristic generalization of the constitutive equation reaches its limits, rapidly. Furthermore, realistic boundary conditions are straightforward for the strain field, but not at all if the stress is used as variable. A critical comparison between the two approaches at the quadratic nonlinear level is given in Ref. [31].

The main results are given in Figs. 2 to 4. In all cases the critical heating rate, η_c, and its corresponding frequency, Ω_c, are displayed as a function of different sets of control and material parameters. In particular, we concentrate the discussion on the influence of the magnetic and viscoelastic properties on those quantities.

Figure 2 shows η_c and Ω_c as a function of the gravity parameter α for five different Deborah numbers. For $\Gamma \neq 0$, we can observe that the system suffers a Hopf bifurcation, which only slightly depends on the Deborah number. For small values of α the dominant regime is the stationary instability, and after the bifurcation the threshold increase its value. In the oscillatory instability regime, the critical heating rate decreases as the gravity parameter increases, while the variation of the Deborah numbers is rather irrelevant. The critical frequency on the other hand increases with α according to a power law and is strongly decreasing with Γ well in the instability

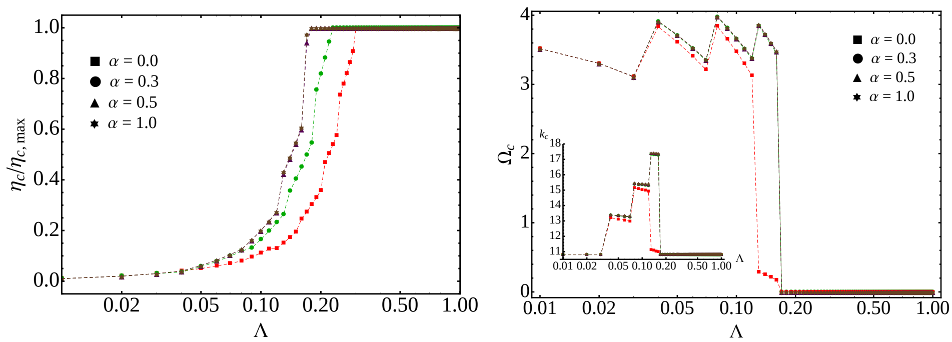

Fig. 4. η_c (left) and Ω_c (right) as a function of Λ for different values of α (red squares $\alpha = 0$, green dots $\alpha = 0.3$, purple triangles $\alpha = 0.5$, brown stars $\alpha = 1$) . The fixed parameters are $P = 10$, $\Gamma = 100$, $M_1 = 10$, $M_3 = 1.1$, $Bi = 10^{-6}$, and $\chi_b = 1$. The inset on Ω_c shows its critical wave number k_c as function α.

regime. Let us remark that the case $\Gamma = 0$, which is the Newtonian case, only the stationary bifurcation appears.

Figure 3 shows η_c and Ω_c as a function of the magnetic number $M_1 \sim H_0^2$ for the same five values of Γ in the pure Bénard case ($\alpha = 1$). We find that the magnetic field destabilizes the system, since the critical value of the threshold, η_c, decreases when M_1 increases. The threshold is independent of the Deborah number Γ. On the other hand, the critical frequency Ω_c is almost independent of the magnetic field. However, it depends strongly on the Deborah number, and decreases for increasing Γ. This simply reflects the expectation that a slower relaxation of the elasticity enhances the internal time scale for the critical perturbations. Notice that the critical heating rate in Newtonian case is lower than the in viscoelastic cases for an intermediate range of M_1, and tends to the same value for both small and large M_1 (see the inset of Fig. 3). Finally, we remark that, for this range of parameters, the influence of the other magnetic number, M_3 is irrelevant, because η_c and Ω_c change less than 0.05%.

Figure 4 shows η_c and Ω_c as a function of the retardation modulus Λ for different values of the gravity parameter α. Obviously, for small Λ there is an oscillatory instability with a (hard mode) transition to the stationary one, with a finite critical frequency at the transition. Such a transition has to be expected, since for a Newtonian fluid ($\Lambda \to 1$) only the stationary instability is possible, while in the elastic case ($\Lambda \ll 1$) convection cannot be stationary, but only oscillatory. This transition occurs at a smaller elastic modulus in the Marangoni case than in the Bénard one. In addition, we observe that in the oscillatory regime the frequency is a non-smooth function of Λ, a known phenomenon in viscoelastic fluids with complex boundary conditions [23,38,39]. It is directly related to jumps in the critical wavenumber k_c (The inset of Fig. 4). At the transition the horizontal width of the spatial patterns (e.g. convection rolls) decreases and reaches again the stationary value in a few steps when decreasing Λ. Finally, let us comment that this phase transition is robust against changes of the Deborah number within a wide range. The critical heating rate is independent of the elastic properties for the stationary instability, but decreases in the oscillatory one rather rapidly with increasing elasticity.

4 Final remarks

In the present work, Bénard-Marangoni convection in a magnetic viscoelastic liquid was studied. The stability thresholds have numerically been determined by the spectral method. The technique of collocation points (Gauss−Lobatto) as described by Threfethen was used [37]. Due to the presence of various destabilizing effects, i.e. buoyancy and magnetic forces, and of additional relaxation channels due to the Oldroyd model, the discussion of the stability curves becomes rather intricate. We found that the magnetic effects destabilizing the system and in the pure Marangoni case its dependence on the critical thresholds is irrespective of low values of the Deborah number. The oscillatory instability, whose critical frequency is a non-smooth function of the relaxation-retardation times ratio, is competing with the stationary one. Let us finally comment that, very often, ferrofluids show a finite separation ratio and a finite magnetic separation ratio and therefore require a binary mixture description. However, for materials where the separation ratio and magnetic separation ratio are not too large the simple fluid approximation is valid [6]. The present work is based on this last approximation. A detailed study on the oscillatory bifurcation for magnetic binary mixtures is still in progress.

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