Localized waves in a parametrically driven magnetic nanowire

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Abstract. - The pattern formation in a magnetic wire forced by a transversal uniform and oscillatory magnetic field is studied. This system is described in the continuous framework by the Landau-Lifshitz-Gilbert equation. We find numerically that, the spatio-temporal magnetization field exhibits a family of localized states that connect asymptotically a uniform oscillatory state with a extended wave. Close to parametrical resonance instability, an amended amplitude equation is derived, which allow us to understand and characterize these localized waves.

Introduction. – During the last years macroscopic particle-type solutions or localized states in dissipative systems have been observed in different fields, such as: domains in magnetic materials, chiral bubbles in liquid crystals, current filaments in gas discharge, spots in chemical reactions, localized states in fluid surface waves, oscillons in granular media, isolated states in thermal convection, solitary waves in nonlinear optics, dissipative solitons in magnetic materials, to mention a few (see the reviews [1–5] and references therein). In one-dimensional spatial systems, localized states can be described as spatial trajectories that connect one steady state with itself, which means, they are homoclinic orbits from the dynamical system point of view [6]. Whereas, domain walls or front solutions are seen as spatial trajectories connecting two different steady states—heteroclinic curves of the corresponding dynamical system [7]. Particular localized states of an extended system in one-dimensions, which has focused much efforts in the past decade, are localized patterns. One can understand these localized patterns as patterns extended over only a small portion of the system [8, 9]. Hence, the localized patterns are homoclinic trajectories that link a uniform state with itself, which passes close to a pattern state [6, 7]. Recently, a geometrical interpretation of the existence, the stability properties, and the bifurcation diagram of localized patterns in one-dimensional extended systems has been proposed [10, 11]; and more recently, the existence of localized patterns based on front interaction was developed in Ref. [12].

On the other hand, for quasi-reversible systems, time reversal systems perturbed slightly injection and dissipation of energy [13], the prototype model that exhibits localized structures (dissipative solitons) and pattern states is the parametrically driven damped nonlinear Schrödinger equation (PDNLS) [14–16]. This model has been derived in several contexts to describe patterns and localized structures like vertically oscillating layers of water [17], nonlinear lattices [18], optical fibers [19], Kerr type optical parametric oscillators [20], magnetization in an easy-plane ferromagnetic exposed to an oscillatory magnetic field [21, 22], and parametrically driven damped pendula chains [23]. However in Refs. [24, 25], we have shown that this model does not account for a family of localized states that connect asymptotically an uniform oscillation with itself. The main reason of the lack of these states in PDNLS is the loss of stability of the uniform, which accounts for uniform oscillation state. The stability of these states is controlled by higher nonlinearities. Hence, in this framework a localized state that connects asymptotically a uniform oscillation with a extended wave can not be expected in this approach—we term this state as *localized waves*. Nevertheless, numerical simulations, in a ferromagnetic anisotropic wire exposed to an oscillatory magnetic field in the validity region of parameter space of PDNLS exhibit these types of localized waves.

The aim of this work is to study the family of localized waves observed in magnetization field of an easy-plane ferromagnetic wire in the presence of a uniform and an oscillatory magnetic external field. Amending the PDNLS with higher order terms, allows us to explain both, the existence and stability of the localized waves, homogeneous oscillations and extended wave solutions. Hence, using this amended amplitude equation, we recover the original dynamical behavior of the magnetic system. Moreover, close to the pattern instability, we are able to characterize this bifurcation as a quintic supercritical bifurcation. The paper is arranged in the following way: In Sec. 2 the theoretical model and the numerical simulations are presented. In Sec. 3 the magnetization dynamics is reduced to a nonlinear oscillator system. The amending amplitude equation is derived and numerical simulations are performed in Sec. 4. Finally, conclusions are given in Sec. 5.

Theoretical Model. – The standard approaches to study the dynamics of the macroscopic magnetization reversal are the Landau-Lifshitz (LL) [26] or the Landau-Lifshitz-Gilbert (LLG) equation [27]. Nonlinear time dependent problems in magnetism have already been studied in many cases, and an account of the state of the art can be found in Refs. [28–30]. These models have been using in both discrete [31–33] and continuous magnetic systems [21, 22, 24, 25].

Let us consider a magnetic wire in the continuous framework, such that the normalized magnetization field is given by $\mathbf{M} = \mathbf{M}(\mathbf{r}, t)$, where $\mathbf{r}$ and t stand for the space coordinates and time, respectively. We focus in a ferromagnetic anisotropic long wire, so we consider the movement of magnetization along the wire axis, represented by $\hat{z} = (0, 0, 1)$. Hence, the dynamical evolution of this wire can be modeled by the LLG equation and it can be written as:

$$\partial_t \mathbf{M} = -\mathbf{M} \times \mathbf{\Gamma} + \lambda \mathbf{M} \times \partial_t \mathbf{M}. \quad (1)$$

The effective torque field, $\mathbf{\Gamma}$, is given by $\mathbf{\Gamma} = \nabla^2 \mathbf{M} - \beta (\mathbf{M} \cdot \hat{z}) \hat{z} + \mathbf{H}$, where the Laplacian term accounts for the coupling of the magnetization with the first neighbors, $\beta > 0$ is the easy-plane anisotropy constant and $\mathbf{H}$ is the external magnetic field. Let us take into account an external magnetic field $\mathbf{H}$ comprises both a constant and oscillatory parts, that is $\mathbf{H} = H_x \hat{x} \equiv (H_0 + h_0 \cos(\omega t)) \hat{x}$. In the above equation, λ denotes the dimensionless phenomenological damping coefficient which is characteristic for the material and whose typical value is of the order 10^{-4} to 10^{-3} in garnets and 10^{-2} or larger in cobalt or permalloy [30].Throughout this manuscript we use dimensionless quantities having scaled the magnetization (and magnetic fields) by the saturation magnetization M_s, the time t by $1/|\gamma| M_s$, where γ is the gyromagnetic factor associated with the electron spin $|\gamma_e|\mu_0$, and the space coordinates $\mathbf{r}$ by the exchange length $l_{ex} = \sqrt{2J/\mu_0 M_s^2}$, where

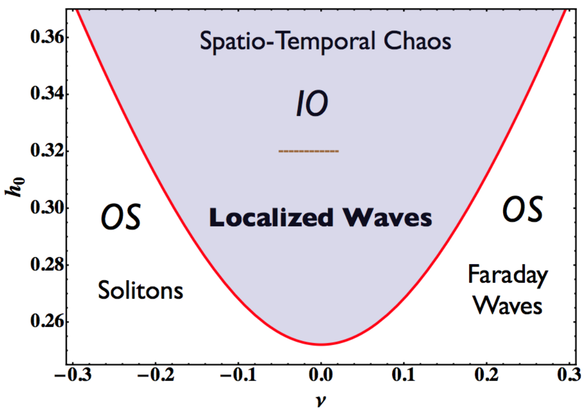

Fig. 1: Phase Diagram of Eq. (1) as a function of ν and h_0. The shaded region is the Arnold's tongue. The dashed line separates the existence region of the localized waves. The fixed parameters are $\lambda = 0.025$, $H_0 = 1$ and $\beta = 20$.

J is the effective exchange coupling constant. Taking, e.g., material values [30] $M_s \sim 800$ kA/m or $\mu_0 M_s \sim 1\,$T, $J = 2\pi \times 10^{-11}$ and $|\gamma| \approx 2.21 \times 10^5$ m A^{-1}s^{-1} the dimensionless time and length scale correspond to ≈ 6 ps and to ≈ 10 nm as physical scales, respectively. The present technology is able to follow experiments at the femtosecond scale. Indeed, Beaurepaire et al. [34] were the first to observe the spin dynamics at a time-scale below the picosecond scale in nickel particles.

Steady states. A simple homogeneous state of model (1) is $\mathbf{M} = \hat{x}$, which represents a uniform magnetization parallel to the magnetic forcing. Small perturbations of this homogeneous state are characterized by damped dispersive waves, with frequency close to $\Omega_0 = \sqrt{H_0(H_0 + \beta)}$. When the wire is forced close to double of this natural frequency, $\omega \equiv 2(\Omega_0 + \nu)$, ν being the detuning parameter, this uniform state becomes unstable by means of a oscillatory instability. This Subharmonic bifurcation is characterized by a Floquet multiplier in the complex space that crosses the unit cycle through -1. This bifurcation gives rise to a uniform attractive periodic solution, which corresponds to a parametric resonance [35]. More precisely, the bifurcation occurs at $h_{0c}^2 = (4\Omega_0)^2 [\nu^2 + (\lambda q/2)^2]/\beta^2$ with $q = \beta + 2H_0$; and this relationship defines the first Arnold's tongue. Figure 1 shows the phase diagram for model (1) for the first Arnold's tongue. We can recognize two different zones, outside (OS) and inside (IS); for instance in the left hand OS-zone appear soliton solutions [21] and in the right hand OS-zone appear extended waves [24]. Also, in the IS-zone spatio-temporal chaos appears [36] as well as oscillating states and localized waves, as we shall see later.

The uniform oscillations account for uniform or synchronized precession motions of the magnetization around the easy axis $\hat{x}$ in the zy-plane with frequency close to Ω_0, as is shown in Figure 2. Note that M_z and M_y have the same frequency, however they are out of phase and M_x has

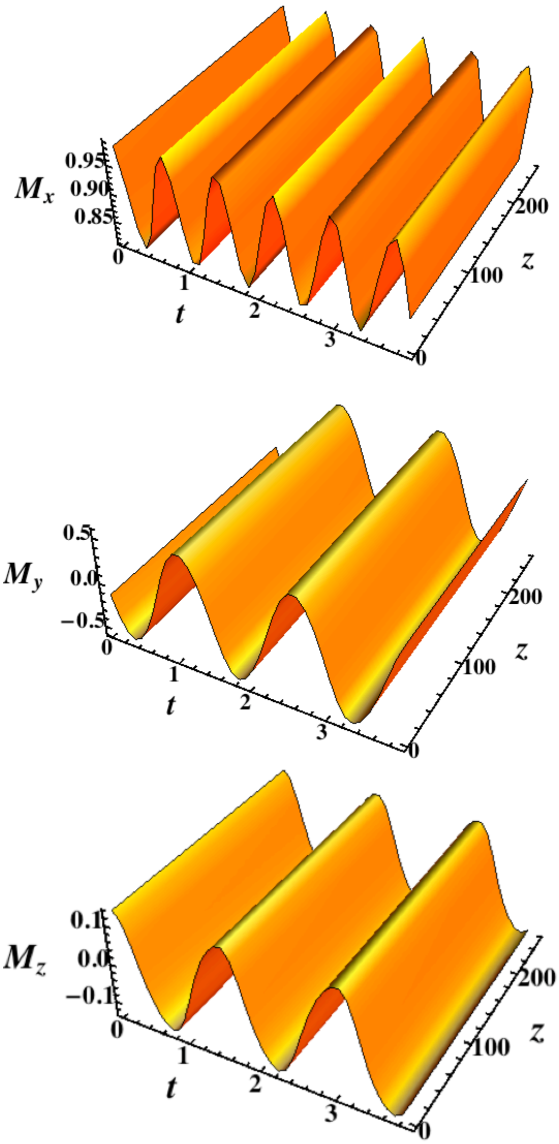

Fig. 2: (color online) *Uniform oscillations:* spatio-temporal diagram of the three components of the magnetization field $\mathbf{M}$ appearing in Eq. (1) by $\lambda = 0.025$, $H_0 = 1$, $\beta = 20$ $\nu = -0.002441$ and $h_0 = 0.28$.

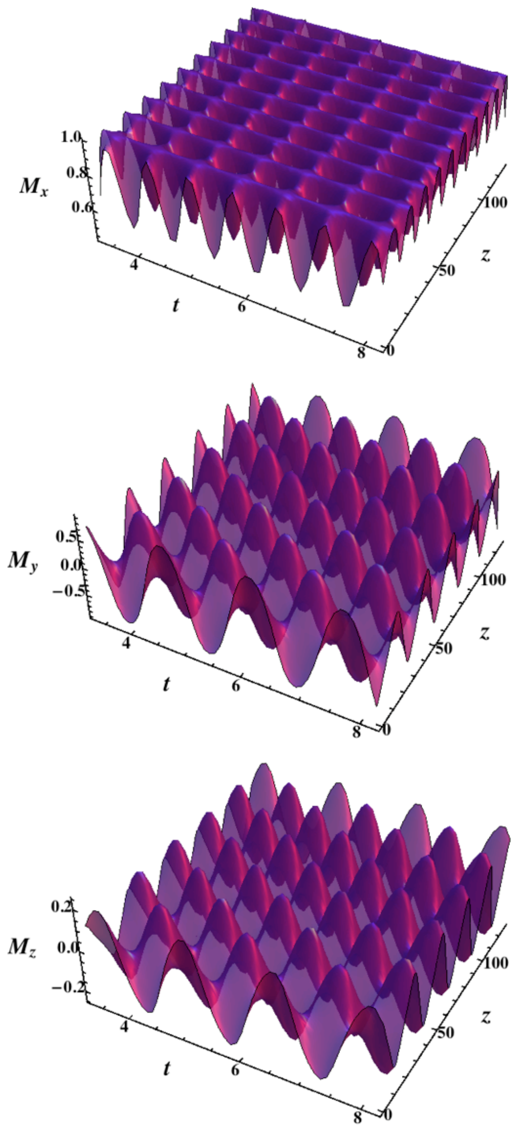

Fig. 3: (color online)*Sub-harmonic extended waves*: spatio-temporal diagram of the three components of the magnetization field $\mathbf{M}$ appearing in Eq. (1) by $\lambda = 0.025$, $H_0 = 1$, $\beta = 20$ $\nu = -0.002441$ and $h_0 = 0.28$.

twice of this frequency. One can understand these dynamical behavior in the following way: $M_x^2 = 1 - M_y^2 - M_z^2$, as a consequence of the conservation of the magnetization modulus. Assuming $M_y \sim M_z \ll 1$, we can approximate $M_x \approx 1 - (M_y^2 + M_z^2)/2 - (M_y^2 + M_z^2)^2/8$. Moreover, M_z and M_y are oscillatory functions around zero with period $T = 2\pi/\Omega$, then M_x is a periodic function with half period.

In the same parameter region, we numerically observe that the system exhibits sub-harmonic extended waves with frequency Ω_0 for M_z and M_y and frequency $2\Omega_0$ for M_x. This motion constitutes a extended wave of the precession motion of the magnetization around the easy axis as shown Figure 3.

Localized waves. Due to the coexistence of these two extended steady states, one expects to find a family of solutions that link these states—localized waves– which is a solution composed of a pair of fronts that corresponds to a homoclinic solution. To our knowledge this is a novel type of localized state for parametric driven systems; and the main ingredient for the existence of this localized states is the coexistence between uniform oscillation and extended waves. We have numerically observed these localized solutions. Figure 4 shows the three component of the vector field $\mathbf{M}$ in its spatio-temporal diagram, where localization is observed in all components. Furthermore, the oscillation frequencies of the components are synchronized. These states are particle-type solutions [6], that is, these states can be created or destroyed in any place of the sys-

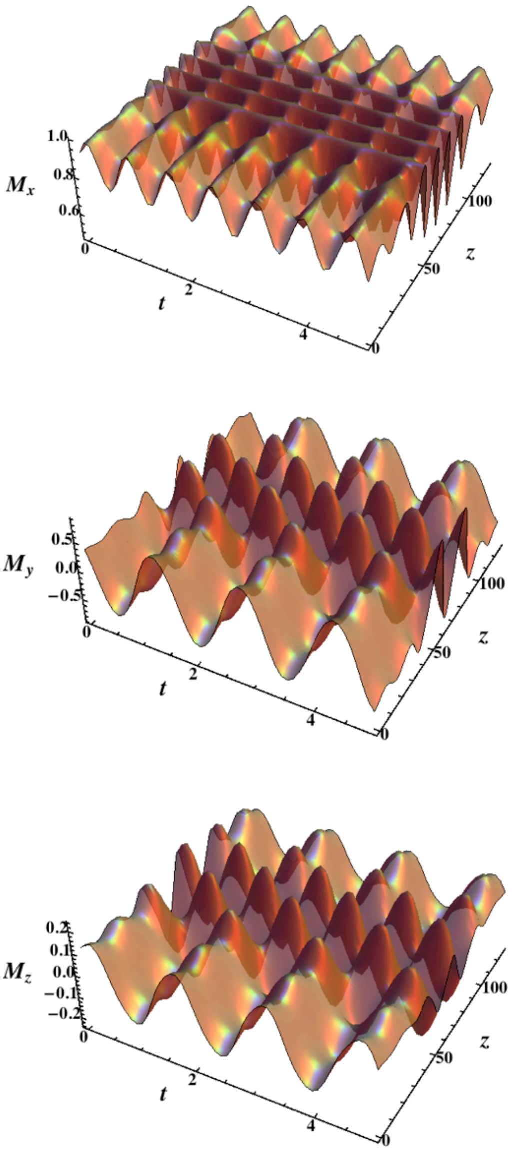

Fig. 4: (color online)*Localized waves*: spatio-temporal diagram of the three components of the magnetization field $\mathbf{M}$ appearing in Eq. (1) at $\lambda = 0.025$, $H_0 = 1$, $\beta = 20$ $\nu = -0.002441$ and $h_0 = 0.28$.

tem. Besides, these states are characterized by a set of parameters: the position of the center of mass and the width [12]. Hence, from this states one can build up a gas or mixture of these localized sates. Finally, we remark that the existence region of localized waves, for a set of fixed parameters, is shown in Figure 1 which is the region bellow to the dashed line.

Amended Amplitude Equation. – Due to the complexity of model (1) only fully numerical solutions are possible. To gain more insight we study these localized states in a second step by an amplitude equation, which is quite simpler mathematically. Close to the parametric instability it can systematically be derived from the full dynamic equations [22] and generally gives a qualitatively correct description, although often quantitative agreement is not obtained. [9]. The standard amplitude equation that describes our magnetic system close to the parametric resonance is the parametrically driven and damped nonlinear Schrödinger equation (PDDNLS) [21]. It is a partial differential equation for the complex amplitude A for the envelope of the oscillations, $m_z(t,z) = A(T,Z)\exp(i(\Omega+\nu)t) + c.c. + \Sigma(A,t)$, where $c.c.$ signifies the complex conjugate and $\Sigma(A,t)$ is a small correction function in the form of a polynomial series in A. One finds after some lengthy calculations the following solvability condition [21]

$$\partial_T A = -i\nu A - i|A|^2 A - i\partial_Z^2 A - \mu A + \gamma \bar{A}. \qquad (2)$$

where $\mu = \lambda(2H_0+\beta)/2$ and $\gamma = h_0(2H_0+\beta)/(4\Omega_0)$ are the effective damping and driving parameters, respectively. Also, $\bar{A}$ stands for the complex conjugate of A and the normalized variables are defined by $Z \equiv \sqrt{2\omega_0/(\beta+2H_0)}z$ and $T = \gamma t$. This model has often been studied to understand soliton like solution [21] and Faraday waves [24, 39]. Notice for certain values of the parameters, model (1) can be approximated by the nonlinear Klein-Gordon equation and from this model one can derive the above equation [21].

In addition, the PDNLS equation has different homogeneous states. The simple one is $A = 0$, which represents the magnetization aligned in the wire direction ($\mathbf{M} = \hat{x}$). Moreover, inside the Arnold's tongue, which is defined by $\gamma^2 \geq \mu^2 + \nu^2$, this model has additional uniform states $A_{\pm,\pm} = \pm x_0(1 \pm i y_0)$, where $x_0 = \sqrt{(\gamma-\mu)(-\nu+\sqrt{\gamma^2-\nu^2})/2\gamma}$ and $y_0 = \sqrt{(\mu-\gamma)/(\mu+\gamma)}$. These solutions stand for a uniform precession around the x-axis along the wire. However, these states are unstable and $A_{+,\pm}$ is marginally stable for zero detuning as consequence of the spatial coupling. Hence, the only uniform linear stable state is $A = 0$. Without the spatial coupling this state is stable outside the Arnold's tongue. The spatial coupling describes by the laplacian term in Eq. (2) modifies this scenario and the zero state exhibits a spatial instability at $\gamma = \mu$ (for positive detuning) giving rise to the occurrence of pattern states. This instability is a quintic supercritical bifurcation [39], i.e. close to the bifurcation the pattern amplitude increases according to the power law $(\gamma-\mu)^{1/4}$. If we decrease the detuning parameter, at fixed $\{\gamma,\mu\}$, the amplitude and the wave number of the pattern are slightly modified. These patterns still occur inside the Arnold's tongue. Nevertheless, close to the boundary of the Arnold's tongue for negative detuning the pattern state disappears by a saddle-node bifurcation.

In brief, the PDNLS exhibits as trivial extended stable state the zero solution and pattern state, which however exist in different parameter regions. Hence, this model does not account for the coexistence of a pattern state and non zero amplitude states ($A_{\pm,\pm}$), which is the minimal ingredient to observe a localized pattern. From the

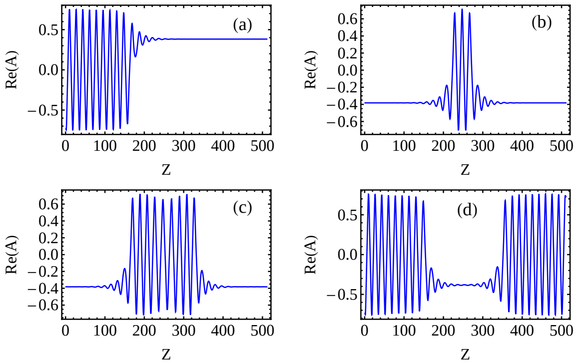

Fig. 5: *Localized patterns, front and hole solutions:* Real Part of the amplitude A for equation (3) as a function of Z at $\gamma = 0.57$, $\nu = -0.002441$, $\mu = 0.275$ and $\gamma\eta = 0.1$.

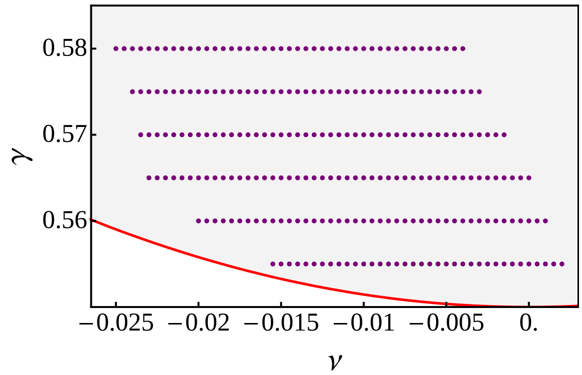

Fig. 6: $\nu - \gamma$ phase diagram of the existence of the localized patterns at $\mu = 0.275$ and $\gamma\eta = 0.1$. The solid line represents Arnold tongue. The gray area and points account for, respectively, the oscillations and extended waves.

dynamical system theory, the prototype model (2) is structurally unstable [35]. The addition of higher order terms can modify the stability features of the non zero amplitude states.

To describe uniform precession and the family of localized states exhibited by the magnetic system under study it is required to consider the high order terms that amend the amplitude equation (2). These new terms also provide the stability of the uniform states and the particle-type solutions. In the parameter region where the non zero amplitude state is marginal at $\nu = 0$, we expect that any small corrections of the amplitude equation can render this state linearly stable or unstable. Consequently, when we consider the leading higher order terms the *amended amplitude equation* reads [24]

$$\partial_T A = -i\mu A - iA\left|A\right|^2 - i\partial_Z^2 A - \nu A + \gamma\bar{A} + \mathcal{N}_A, \quad (3)$$

such that the amended terms, $\mathcal{N}_A$, are given by

$$\mathcal{N}_A = -\alpha A\left|A\right|^2 + \kappa\partial_x^2 A - \delta\left|A\right|^2\bar{A} + \tilde{\beta}A^3 + i\eta\left|A\right|^4 A \quad (4)$$

The above terms are order $\gamma^{5/2}$. Also, $\{\alpha, \kappa, \delta, \tilde{\beta}\}$ are complex functions of the parameters [24]. The extra terms add novel behavior, which can be interpreted in the following way: the term proportional to $\{\alpha, \kappa, \delta, \tilde{\beta}, \eta\}$ are respectively a nonlinear dissipation, diffusion, nonlinear parametric forcing and higher nonlinear response in frequency. It is important to note that we have included all extra terms that appear of order $(\mu^{5/2})$, considering a similar injection and dissipation of energy, more precisely we consider the scaling $\nu \sim \gamma \sim \alpha \sim \kappa \sim \delta \sim \tilde{\beta} \sim \mu$, $\partial_T \sim \mu$, $\partial_Z \sim \mu^{1/2}$, $\eta \sim 1$ and $\mu \ll 1$. Equation (2) is of order $\mu^{3/2}$. Since $A_{+,\pm}$ is marginally stable of model (2), the inclusion of extra terms recover the dynamical behavior of the original model (1). Considering only some of these extra terms is sufficient to recover the qualitative dynamics [40], however for consistency reason we have considered all corrections of same order. In Ref. [40] the PDNLS equation is amended by a extra diffusion term, such that this amendment produces novel localized solutions that connect the zero state with a flat nonzero state. A complete study of consequence of extra terms will be presented in future works.

Numerical Solutions. For small detuning, we numerically observe that the amended amplitude equation (3) has stable uniform solutions close to $A_{+,\pm}$ [24,25]. In the next subsection we shown numerical solutions of the amended amplitude equation.

Recently, we have shown that the amended amplitude equation (3) has stable non zero uniform solutions [24], hence the spatial connection between these states with itself or with other state is observed. On the other hand, model (3) presents coexistence between stable pattern and stable non zero uniform states for small detuning, thus this model shows a solution that link these states, which corresponds to a front solution (cf. Fig. 5a). From this elementally solution using the theory of fronts interaction, one can build up a family of localized patterns or hole solutions [12]. The frames (b) and (c) of Fig.(5) show two localized patterns with three and nine bumps, respectively. These solutions represent localized waves in the parametrically driven magnetic system, as those shown in Fig. 4. Therefore, the incorporation of leading high order terms in the parametrically driven damped nonlinear Schrödinger equation accounts for particle-type solutions that link one homogeneous state with a pattern state, producing a localized pattern.

It is worthy to note that in the same region of parameters other types of solutions can be found as shown in Figure (5)d. In this figure, we observe a hole solution, this state being the *inverse solution* of the localized pattern, that is, this solution represents a localized uniform precession surrounded by extended waves.

Finally, in order to study the robustness of the new types of solutions we are numerically calculated the range of parameters where the localized patterns exist. Figure (6) shows the region of coexistence between the uniform oscillations, which exist within the Arnold tongue, and extended waves, which are represented by points .We can observe that the localized state persist in a wide range of parameters.

Final Remarks. – Far from equilibrium systems in general exhibit multistable states. The spatial connection of these states gives rise to a wealth and unexpected dynamical behavior. One of the main goals of nonlinear science is to understand this complex behavior. We have here presented a novel type of localized states that link asymptotically a homogeneous precession state with a sub-harmonic wave in a magnetic anisotropic wire in the presence of a parametric external magnetic field in the transverse direction. The conventional approach of this system lacks this novel family of localized pattern states. The improvement of this model by consideration of leading high order terms account for these localized states in a unified manner. Hence, due to the universal nature of the considered model we expect to observe localized waves in several driven systems such as a vertically oscillating layer of Newtonian fluid, forcing nonlinear lattices, optical fibers and Kerr type optical parametric oscillators.

Acknowledgments

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