Macroscopic Dynamics and Hydrodynamic Maxwell Equations

In an interesting and stimulating Letter[1], it has been suggested that the conventional Maxwell equations of continuous media are incomplete and must be replaced by *hydrodynamic* Maxwell equations.

Here we point out

a) that the additional terms suggested in ref.1 are not associated with truly hydrodynamic variables; and

b) that there is no apparent general evidence that the suggested terms are more relevant for the description of macroscopic processes than all the other (typically 10^{23}) microscopic variables.

Truly hydrodynamic variables come in two groups[2,3]: (i) conserved quantities such as mass density, energy density and density of linear momentum for simple fluids and many other systems; and (ii) variables associated with spontaneously broken continuous symmetries such as for example the staggered magnetization in an antiferromagnet (associated with broken rotational symmetry in spin space). All truly hydrodynamic variables are connected with a hydrodynamic collective excitation whose frequency ω vanishes in the long wavelength limit:

$$\lim_{k\to 0} \omega(k) = 0.$$

Clearly, of the Maxwell equations the charge conservation law (related to $\nabla \cdot \mathbf{D}$) belongs to group (i) and no part belongs to (ii).

However, there are systems or special circumstances, where it is necessary to consider another class of variables, namely macroscopic variables. The excitation frequency of modes associated with these does not vanish in the long wavelength limit $\lim_{k\to 0} \omega(k) \neq 0$, but their lifetime is sufficiently long so that they become important for excitations of finite wavelength. To take into account such macroscopic variables is a valid description only, if they are much slower than, and thus clearly distinguishable from all the microscopic variables, which relax on microscopic timescales associated with the average time between two collisions of the constituent atoms or molecules. Macroscopic variables include, for example, the modulus of the order parameter near a second order or a weakly first order phase transition and this quantity has been incorporated into the macroscopic dynamics near the superfluid λ-transition in ^{4}He by Khalatnikov[4]. The concept of macroscopic variables has since been applied to many other condensed matter systems including superfluid ^{3}He, incommensurate crystals and various liquid crystalline phases[5−8].

In ref.1 additional dissipative terms proportional to the two additional thermodynamic forces $\nabla \times \mathbf{H}$ and $\nabla \times \mathbf{E}$ are added to the dynamic equations for $\mathbf{D}$ and $\mathbf{B}$. This means that the non-hydrodynamic variables $\nabla \times \mathbf{B}$ and $\nabla \times \mathbf{D}$ are introduced as macroscopic variables.

First we note that $\mathbf{D}$ is not a hydrodynamic variable. This can be seen from the first of the equations in eq.(2) of ref.1, where the term proportional to the electric conductivity contains no gradients thus demonstrating that $\mathbf{D}$ cannot be truly hydrodynamic. While $div\mathbf{D}$ is conserved (charge conservation), $curl\mathbf{D}$ must be non-hydrodynamic. Therefore, in a hydrodynamic description $curl\mathbf{D}$ and $curl\mathbf{B}$ – as all other nonhydrodynamic variables – are assumed to relax on an infinitely short time scale, and only $div\mathbf{D}$ is kept as (hydrodynamic) variable.

From the discussions leading up to eq.(7) of ref.1 one concludes that the newly introduced transport coefficients α and β are associated with relaxation phenomena and not with hydrodynamic excitations. Thus these newly introduced coefficients are connected with non-hydrodynamic effects and one would have to argue why $curl\mathbf{D}$ and $curl\mathbf{B}$ should be kept in the list of macroscopic variables. Otherwise, $\nabla \times \mathbf{D}$ and $\nabla \times \mathbf{B}$ relax on a microscopic time scale and are thus adiabatically eliminated from hydrodynamics as all the other microscopic variables.

Finally we stress that in eq.(9) of ref.1 the electric conductivity has been assumed to be identical to zero. Taking any non-vanishing value - as is the case for a l l realistic physical systems - it becomes clear that eq.(9) is changed qualitatively giving rise to a relaxation due to σ.

Thus unless the suggested dissipative coefficients α and β of ref.[1] are sufficiently large for some very special systems or circumstances, there is no need to change the classical Maxwell equations of continuous media.

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