Temmen et al. Reply: The criticism of the preceding Comment [1] is based on misunderstanding and misrepresentation of our recent Letter [2], and is therefore mistaken.

In this Letter, we unambiguously derived the convective nonlinearities in the dynamic equations, both for any elastic media and for the elastic degrees of freedom in the transient elasticity of viscoelastic fluids. Most frequently, these convective nonlinearities are derived by postulating *ad-hoc* principles, especially the *material frame-indifference principle*. This traditional approach contains two basic flaws. First, it does not lead to a unique form of the convective nonlinearities, with the remaining gap usually bridged by a completely arbitrary choice of the authors. The result is then, characteristically and euphemistically, referred to as a *model* – such as *upper* or *lower convected*, or *corotational*.

Second, and more suspicious, the principle does not lead to a well defined procedure: Stated in the phrase that material properties are independent of the state of the observer, it is a trifle and trivially true, but quit useless. Implemented as invariance of all *constitutive equations* under arbitrary time-dependent coordinate transformations, it is much too confining – unless one starts to quibble about what a *constitutive equation* is, reducing the *frame-indifference principle* in effect to a tautology – a constitutive equation is what satisfies the frame-indifference principle. For instance, as Newton's equation changes its form if written in a rotating frame, it is not one. Not surprisingly, discussions about what constitutes a constitutive equation are a popular pastime among people who subscribe to the frame-indifference principle.

Isotropy of space and Galilean invariance are the most general nonrelativistic invariance principles in physics. The first is satisfied as long as we adhere to the tensor notation, the second delivers constraints on terms containing the velocity, similar to the frame-indifference principle. The important difference, however, is that Galilean invariance is a genuine first principle, and it delivers unique results, especially Eq (7) of [2], from which all convective nonlinearities follow by simple algebra.

Our results show that the "covariant convected" form holds for the Eulerian strain tensor, to arbitrary order in the strain, yet does not for other possible variants of the strain tensor, e.g. the invariant strain tensor, cf [3]. Here, the convective nonlinearities are markedly different from any lower or upper convected form. In fact, they involve all powers of the strain. This result clearly demonstrates that the convective nonlinearities cannot be postulated but need to be derived. (Although we referred to the "covariant" as "upper", and to the "contravariant" as "lower" convected derivative in [2], the reverse identification is in fact more usual.)

Now our rebuttal of some more specific criticisms of the preceding Comment. First, as pointed out above, the convective nonlinearities follow from the dynamics of the proper hydrodynamic variable as discussed in detail in [2]. To the best of our knowledge, this dynamics has not been given before – it is certainly not in reference 1-4 of [1], as claimed.

As already mentioned in [2], in many systems the description of visco-elasticity requires additional relaxing degrees of freedom if they are sufficiently slow. On this point we agree with the authors of [1]. What they however do not seem to realize is that the introduction of additional degrees of freedom does not in any way change the form of the convective nonlinearities of the elastic degrees of freedom. That is why we were able to restrict our considerations to the elastic degrees of freedom and still draw general conclusions.

The general case of anisotropic elasticity was discussed in [2], because there are anisotropic visco-elastic fluids, including liquid crystalline polymers. For the isotropic case, the rotation variables drop out automatically.

Visco-elasticity is by definition a phenomenon in which the elastic behavior is not static or permanent, but relaxes in a finite time. This time discriminates "short" from "long," and is material dependent. Yet "short" of course always implies times within the macroscopic scale. One must not get confused here.

The irreversible dynamics of solids may be quite different from the short-time irreversible dynamics of visco-elastic fluids, but no irreversible effect can change the form of the (reversible) convective nonlinearities. Thus, differences in the irreversible dynamics do not prevent us from using the elasticity of an elastic medium for deriving the convective nonlinearities for the elastic degree of freedom in visco-elastic fluids.

In conclusion, we question the usefulness and trustworthiness of the material frame-indifference principle and have demonstrated in the case of the elastic degrees of freedom in visco-elastic fluids how it can be replaced by derivation employing genuine first principles.

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